

Supermassive Black Holes in Galactic Nuclei

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ABSTRACT

We discuss the link between the observations of distant quasars and those of massive dark objects in the cores of many local galaxies. We show how the formation of early black holes gives rise to the luminosity function of high z quasars, while it imprints into their dark local relics a related shape of the mass-dispersion correlation. We propose that in its lower section the correlation slope will tell the (otherwise uncertain) strength of the feedback effects from the quasar radiation on the host galaxies.

Subject headings: Black hole physics – galaxies: active – galaxies: interactions – galaxies: nuclei – quasars: general

1. Introduction

Two news have recently kindled the field of the quasars and active galactic nuclei.

The first concerns the farthest objects. Not only single quasars (Qs) have been detected at redshifts out to $z = 6.28$, but also the statistics at $z \approx 5$ has improved to the point of outlining the bright section of the luminosity function (Fan et al. 2001a, 2001b).

These findings confirm that the population of the optically selected objects goes through the most sharp and non-monotonic of evolutions. The comoving density of the bright sources rises on the scale of a few Gyrs from the Bang, to peak at around $z \approx 3$ (Shaver et al. 1996); later on it turns over and falls by factors 10^{-2} toward us (Boyle et al. 2000). A similar message comes from the radio band, see Jackson & Wall (1999).

The second piece of news concerns the cores of many local galaxies, where massive dark objects (MDOs) ranging from a few 10^6 to a some $10^9 M_\odot$ had been detected within regions from a few to tens of pcs, see Richstone et al. (1998). Now (Ferrarese & Merritt 2000; Gebhardt et al. 2000) such masses are found to correlate tightly with the ve-

locity dispersion in the body of the surrounding host galaxies.

The MDO masses fit into the framework provided by the long standing arguments (see Lynden-Bell 1969) that indicate gravitational contraction rather than thermonuclear burning to be the dominant source of QS output. We just recall this conclusion to hinge upon the high bolometric power $L > 10^{45} \text{ erg s}^{-1}$ of many such sources, and on the high compactness with sizes down to $R \sim 10^{15} \text{ cm}$ indicated for some of them; it also requires an overall efficiency up to $\eta \sim 10^{-1}$ for conversion of gravitational into radiative power.

But then the argument may be carried on to evaluating the masses involved (Cavaliere et al. 1983); in terms of $M_8 = M/10^8 M_\odot$ these read

$$M_8 \approx 3 (L_{45} \Delta t_{-1} R_{15} / \eta_{-1})^{1/2}, \quad (1)$$

where we have used $L_{45} = L/10^{45} \text{ erg s}^{-1}$, $R_{15} = R/10^{15} \text{ cm}$, and the source life time $\Delta t = \Delta t_{-1} 10^{-1} \text{ Gyr}$. Such masses are consistent with the MDO observations.

Here we intend to link the largest values of M with the early Qs, and to discuss how such a relation is to be extended into the range of lower redshifts and smaller masses.

2. Black holes and their environment

The accreting black hole (BH) paradigm not only best accounts for the small sizes and the top efficiencies required, but also provides to the spent masses the stability of a terminal configuration (Rees 1984). Nailing down the indication from eq. (1), a BH keeps full record of the mass

$$M = \int dt L(t)/\eta c^2 \quad (2)$$

accreted over the life of an active galactic nucleus.

Meanwhile, the induced bolometric luminosities

$$L \approx \eta c^2 \Delta m / \Delta t \quad (3)$$

are tuned (even at constant $\eta \sim 10^{-1}$) by the mass Δm accreted over a time Δt , and cover a wide dynamic range: from Eddington *self-limiting* conditions governed by radiation pressure that yield $L \sim L_E \approx 10^{46} M_8 \text{ erg s}^{-1}$, to *supply-limited* accretion (Cavaliere & Padovani 1989) that easily allows only sub-Eddington emission.

Thus strong gravity is not enough. Equally important are the *environmental* conditions in, or surrounding the host galaxy; these can drive widely ranging accretion rates from 10^{-3} or less, up to some $10^2 M_\odot \text{ yr}^{-1}$. In addition, it is the cosmological change of the environment that qualifies to govern the QS evolution; in fact, both occur on time scales $t_{ev} \sim$ a few Gyrs such that $\eta t_E \ll t_{ev} \ll H_0^{-1}$ holds.

In turn, the environmental conditions are described by the other paradigm, the hierarchical growth and clustering of dark matter (DM) halos, wherein the galaxies constitute lighter baryonic cores (White & Rees 1978). This implies substantial dynamical events to occur, at early z in the strong form of merging between comparable subgalactic units, later on as milder interactions of galaxies in groups.

All these dynamical events tend to break on scales of kpcs the axial symmetry of the galactic gravitational potential, or enhance its steady asymmetry; relatedly, the specific angular momentum j providing support to the gas in the central kpc of the host is not conserved, rather it is transferred to the massive DM component. Thus the necessary condition is provided for destabilizing and funneling inward a sizeable gas fraction. At

smaller scales dissipative processes take over to redistribute j (Haehnelt & Rees 1993), and cause the gas to reach the nuclear accretion disk and grow new BHs or refuel the old ones.

To link BH growth and QS luminosities we follow Cavaliere & Vittorini (2000) and disentangle the triggering dynamical events into two main *regimes*, roughly divided by the epoch of group formation $z_G \simeq 2.5 \pm 0.5$, depending on cosmological/cosmogonical parameters.

3. QSs in forming spheroids for $z > 3$

The *self-limited* regime occurs mainly at epochs before z_G , when galactic spheroids are built up through major merging events between halos with $M_h < 10^{13} M_\odot$. These events destabilize large amounts of gas, but also replenish the host structures with fresh supplies and sustain the gas amount at the cosmic level $m \approx 10^{-1} M_h$.

As a consequence, central BHs can form and/or accrete rapidly, growing by $\Delta m \sim M$ over dynamical times close to the Salpeter scale, $t_{dyn} \sim \eta t_E$; so after eq. (3) they attain Eddington luminosities $L \sim L_E \propto M$.

In turn, M is related to M_h ; two specific models bracket the processes involved, see Cavaliere & Vittorini 1998 (CV98), Hosokawa et al. (2001). Haehnelt & Rees (1993) considered BH coalescence in parallel with merging of their halos; this process (hereafter model A) is described by the simple scaling

$$M \approx 10^{-4} M_h . \quad (4A)$$

Alternatively, Haehnelt, Natarajan & Rees (1998) proposed the feedback-constrained model B where the scaling reads

$$M_8 \approx (1+z)^{5/2} M_{h,13}^{5/3} . \quad (4B)$$

This is because during halo merging a central BH may also accrete gas up to the limit

$$\epsilon L_E t_{dyn} \lesssim G M_h m / r ; \quad (5)$$

this is set by gas unbinding from the halo potential well, due to the deposition of an (uncertain) fraction $\epsilon \sim 10^{-2}$ of the QS radiation (Silk & Rees 1998).

With both models (4A) and (4B) the early QSs are expected to grow in average luminosity and in

number, tracking the development of protogalactic halos over the range from $M_h \sim 10^{10}$ toward $10^{13} M_\odot$. The evolving halo mass distribution is widely taken in the form $N_{PS}(M_h, z)$ first proposed by Press & Schechter (1974); the positive term of its time derivative provides the rate of halo formation, and yields (see CV98) the luminosity function (LF) in the form

$$N(L, z) dL = \Delta t \partial_t N_{PS}(M_h, z) dM_h, \quad (6)$$

with the prefactor $\Delta t \approx \eta t_E$ accounting for the limited source lifetime.

Fig. 1 shows the optical LFs provided by such models. We stress that model (4B) by the very means of its non-linear transformation stretches the halo distribution into flatter, more fitting LFs and predicts a stronger (negative) evolution. By the same token, it also associates bright QSs with the largest galactic halos, consistent with the data discussed by Hamilton, Casertano & Turnshek (2000).

Both models are normalized to the data at $z \approx 4$. But model (4B) privileges the upper halo range where $N(M_h)$ decreases steeply, so it provides BH numbers smaller and naturally close to *one* large BH per actively star forming protogalaxy of intermediate (Steidel et al. 1999) or large mass (Granato et al. 2001).

In either model, the early mass distribution $N(M, z)$ is directly related to $N_{PS}(M_h, z)$; the model (4B) yields the result represented by the thick solid line in fig. 2.

4. QSs in interacting galaxies for $z < 3$

After $z \approx z_G$ the galaxies are assembled into small groups of mass $M_G \gtrsim 10^{13} M_\odot$, where the dominant member recurrently interacts with its companions, to the effect of refueling and rekindling an old BH; growing evidence (referenced in CV00) relates many QSs with interacting hosts. Small groups, with their high galaxies density and low velocity dispersion V still close to the galaxian dispersion σ , constitute preferred sites for such interactions to occur.

Supply-limited accretion prevails here, since now the gas mass m in the host is depleted but no longer replenished by the interactions. Still, considerable fractions of the gas initially orbiting in the host at $r \sim \text{kpc}$ are destabilized and partly

made available for accretion; such fractions are easily evaluated (see CV00) in the form

$$f_d \lesssim \Delta j/j \approx G M'/\sigma V b. \quad (7)$$

This includes the host structural parameter j/r , taken to be close to σ ; it also includes encounter orbital parameters: the impact parameter b , the relative velocity V , and the partner mass M' . Truly tidal interactions imply a postfactor r/b .

With $V \gtrsim \sigma$ and b bounded by the group radius, interactions and gas inflow both take times $\Delta t \approx b/V \sim 10^{-1}$ Gyr. The above equation easily yields f_d in excess of a few %, of which about 1/3 may reach the nucleus while the rest is likely to end up in circumnuclear starbursts, see Sanders & Mirabel (1996), CV00. This is enough to produce (see eq. 3) outputs $L \gtrsim 10^{46} \text{ erg s}^{-1}$ in a host still gas rich with $m \sim 10^{10} M_\odot$. Fractions up to $f_d \approx 1/2$ are indicated by eq. (7), and in fact obtain in numerical simulations of grazing collisions, which are statistically fewer. The full probability of accreting a fraction f is calculated by CV00 to read $P(f) \propto f^{-2}$ on the basis of eq. (7); this defines range and shape of the LF.

In time, many small groups merge into richer ones where interactions are less frequent and effective. Their volume density, grown rapidly later than z_G , at low z goes down into a demise $N_G(z) \propto (1+z)$; in addition, the decreasing interaction rate $\tau_r^{-1}(z) \approx 0.5 (1+z)^{3/2} \text{ Gyr}^{-1}$ lowers the number of activated sources. The result for the QS population is a moderate “density evolution” proportional to $N_G(z) \tau_r^{-1}(z)$. But a stronger “luminosity evolution” occurs since on average $L \propto f m(z)$ holds, and the residual host gas $m(z)$ is depleted on a scale t_{ev} as it is destabilized and used up in accretion episodes and by accompanying nuclear starbursts, with no replenishment.

As L decreases and M increases, the sources on average go toward sub-Eddington luminosities, consistent with the data in Salucci et al. (1999), Wandel (1999).

5. Relics at $z \approx 0$

For early BHs growing at $z > 3$ the models (4A) or (4B) imply different scaling relations, that read

$$M \propto \sigma^3 \rho^{-1/2}(z) \propto \sigma^4, \quad (8A)$$

or

$$M \propto \sigma^5, \quad (8B)$$

respectively. These stem from the simple hierarchical scaling of $M_h \propto V^2 R/G$ and $R \propto (M_h/\rho)^{1/3}$ for the surrounding DM halos, as is appropriate for $M_h < 10^{13} M_\odot$ when one galaxy per halo occurs. In such conditions, it is also fair to assume $\sigma \propto V$, and to fix the z -dependent fuzz appearing only in eq. (8A) on relating the density to M_h by means of $\rho \propto M_h^{-1/2}$, that holds for hierarchically formed halos (see Haehnelt & Kauffmann 2000).

The relations (8A), (8B) turn out to be in tune with the current debate concerning the MDO data. These have been recently recognized to follow a tight and steep correlation; the precise slope is given as slightly flatter than $M \propto \sigma^4$ by Gebhardt et al. (2000), or close to $M \propto \sigma^5$ by Ferrarese & Merritt (2000). In either case the scatter is found to be rather small, that is, factors $10^{\pm 0.35}$ or less in M at given σ . Note from fig. 1 that LF's flat as observed at high z obtain from model (4A) only upon convolution with scatter larger than $10^{\pm 0.5}$, as discussed by Haiman & Loeb (1998).

Later than $z \approx 3$ fewer new BHs are still produced, but the early ones still grow after $M(z) = M(3) + \sum_i \Delta m_i$, due to additional but dwindling accretion events Δm_i caused by host interactions.

The latter cause the mass distribution $N(M, z)$ to evolve as given by

$$\partial_t N = \frac{N_G}{N_{tot} \tau_r} \int df P(f) [N(M - fm) - N(M)]. \quad (9)$$

Following up §4, $N_G(z)/N_{tot}$ is the fraction of host galaxies residing in a group, relative to total BH number including the dormant ones; for it we take values of order 10^{-1} as discussed in CV00, based on the fraction 1/3 of bright galaxies found by Ramella et al. (1999) to reside in groups with membership 3 or larger. The right hand side describes the net change at z of the distribution $N(M, z)$, due to an accretion episode of $\Delta m = f m(z)$ which upgrades the initial mass $M - \Delta m$ to the current value M ; the probability for this to occur is $P(f)$, and the factor provided by the interaction frequency $\tau_r^{-1}(z)$ converts it to a rate.

The numerical solution plotted in fig. 2 shows how $N(M, z)$ – starting from the condition at

$z = 3$ computed in §3 – drifts and diffuses in time toward higher M , but only up to a cutoff; this is due to the large values of f being rare, and to $m(z)$ being depleted from its initial value $10^{-1} M_h$. So the additions $\Delta m = f m(z)$ decrease on average, and large M are unlikely to grow much larger.

We derive the corresponding $M - \sigma$ relation noting from eq. (7) that the gas masses available for accretion follow $\Delta m = f_d m/3 \propto m/\sigma$. On adopting the scaling $m(\sigma) \propto \sigma^4$ indicated by the Faber-Jackson relation or produced by stellar feedback, the result is $\Delta m \propto \sigma^3$. If used in full, such masses dominate at low $\sigma < 200 \text{ km s}^{-1}$, to yield maximal M values scaling as

$$M \propto \sigma^3, \quad (10)$$

see fig. 3. The scatter is within an overall factor 5, the effective range given by the steep form of $P(f)$. Truly tidal interactions (see the comment under eq. 7) yield a somewhat steeper scaling $\Delta m \propto m(\sigma) r/\sigma \propto \sigma^4$ and more scatter.

But – given that enough gas is made available by the interactions as shown in §4 – the mass actually accreted may still be *constrained* by the QS feedback, depending on the degree of coupling of the source output to the surrounding baryons. Here we focus on the 90% radio quiet sources where the output is mostly radiative and roughly isotropic. At low z the unbinding constraint similar to eq. (5) reads

$$L \Delta t \approx \eta c^2 \Delta m \lesssim \epsilon^{-1} m(z) \sigma^2. \quad (11)$$

With $\epsilon \sim 10^{-2}$, the masses accreted in a host with $\sigma < 200 \text{ km s}^{-1}$ are constrained to stay under those made available by interactions and given by eq. (7); using again $m \propto \sigma^4$, here the result is $\Delta m_i \propto \sigma^6$. These moderate additions drive considerably less evolution of $N(M, z)$ as represented by the thin solid line in fig. 2; then M is always dominated by the initial values which follow eq. (8B), and will in fact converge to values scaling as $M \propto \sigma^5$.

The full scaling laws given by eqs. (8B) with (11), or by (8A) with (10), are represented in fig. 3. They differ mostly in the lower left section, where data are difficult to obtain from the kinematics of stars or of a nuclear disk.

6. Conclusions and discussion

We conclude that the relic BH masses are related to the host galaxy dispersions by the *steep* correlation $M \propto \sigma^5$ if QS feedback onto the host's gas always provides the main constraint to the accretion. In other words, this form is to hold at all σ if the host potential well *bounds* the actual accretion rates all the way from early BH formation to later additional growth.

On the other hand, if such a control was never the limiting factor we expect the softer (and fuzzier) correlation $M \propto \sigma^4$ to hold at high σ , and this to *soften* yet at lower σ to the form $M \propto \sigma^3$ set by the production rate of the available gas.

These two extremes *bracket* the QS and BH story. The latter is centered on one kind of engine, the BH based on strong gravity; but it comprises different regimes of fueling, that is, of accretion and growth triggered by the environment under weak gravity.

At early $z \gtrsim 3$ much fresh gas is made available for full, self-limiting accretion by major merging events which build up the halos and the embedded spheroids. BHs grow rapidly, generating the upper section of the correlation $M \propto \sigma^4 \div \sigma^5$. Meanwhile, the associated QSs flare up at Eddington luminosities; their LF grows in range and height, tracking the progressive halo build up.

Later than $z \approx z_G \approx 2.5$ the dearth of gas curbs the accretion and bends down the QS evolution, as host interactions with group companions are still able to trigger accretion but no longer to import fresh gas. Though supply-limited, the gas masses made available for accretion are still sizeable. If these are used in full, the average luminosities are still high and the BHs still grow considerably; relatedly, the correlation is softened to $M \propto \sigma^3$ in its lower section.

At z lower yet, the host gas reservoirs approach exhaustion, and weaker AGN activity may be sustained by lesser gas productions or smaller supplies provided, e.g., by internal instabilities (see Heller & Shlosman 1994, Merritt 1999) or by satellite galaxies cannibalized by the hosts (CV00). These latest and smaller accretion events can fuel many weaker AGNs more likely to be pinpointed in X-rays, implying widespread accretion power but moderate growth of individual BHs, as also argued by Salucci et al. (1999). Deep surveys

in hard X-rays are now uncovering optically hidden, and even X-ray obscured accretion power, as they resolve and count the AGNs that dominate the hard XRB (see Giacconi et al. 2001, Hasinger et al. 2001); the indications are currently favoring AGNs of intermediate powers and redshifts as main XRB contributors (see Comastri et al. 2001). These findings jointly with the scanty evidence of, and the indirect bounds to a QS 2 population (see Maiolino et al. 2001) suggest to us that the optical-IR sampling is fair to within a factor 2 as for the outputs of early and luminous QSs, and is reliable for linking these with the largest BHs.

Thus our conclusions may be rephrased in terms of two simple *patterns* standing out of this complex picture.

First, the largest MDOs in local inactive galaxies are directly *linked* with the high- z luminous QSs. In particular, we have shown that *flat* optical LFs given by the feedback-constrained model (4B) are linked to the *steep* and *tight* correlation given at large masses by $M \propto \sigma^5$ (see eq. 8B). Current evidence concerning high brightness of the hosts (see §3), flat shape of the high- z LFs (§3), and narrow scatter in the $M - \sigma$ correlation (§5) concur to favor this specific link. But telling data will be provided by the precise slope of the upper MDO correlation observed locally in the optical band, jointly with the rate of QS decline toward high z derived from large area surveys out to the near-IR like the Sloan Digital Sky Survey.

Second and concerning the lower region of the $M - \sigma$ plane, it is conceivable (within the present uncertainties about ϵ) that the fuel throttle to the BH engine is always controlled by the host wells as is expressed by eq. (11); if so, the main accretion modes may be effectively united by the feedback action. In §5 we have shown that this provides the *lower* bound to M expressed again by $M \propto \sigma^5$. On the other hand, an *upper* bound $M \propto \sigma^3$ is set by the condition of maximal fuel availability, eq. (10); this includes the production of optically obscured accretion power. Note that the highly obscured, wind blowing BHs suggested by Fabian (1999) would be located in between. The actual feedback action will be observationally probed by the slope of the $M - \sigma$ correlation in its lower section; here the growing reverberation map database (see Ferrarese et al. 2001) in current AGNs will be telling.

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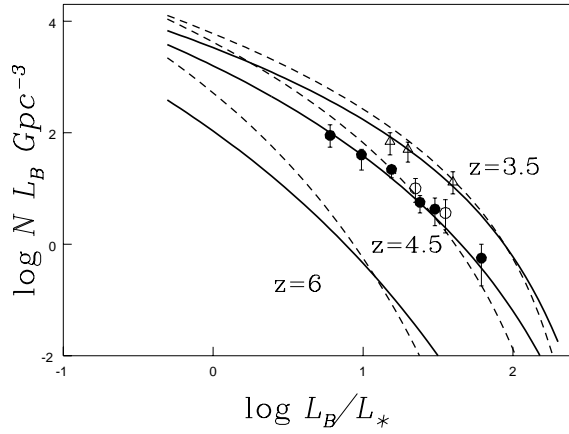


Fig. 1.— High- z LFs from the models discussed in §3: eq. (4A), dotted; eq. (4B), solid. Here L_B is the luminosity in the blue band (bolometric correction $\kappa = 10$), and $L_* = 10^{45} \text{ erg s}^{-1}$; canonical Λ CDM cosmology/cosmogony. Data: filled circles from Kennefick et al. 1995, Schmidt et al. 1995; open symbols from Fan et al. 2001a.

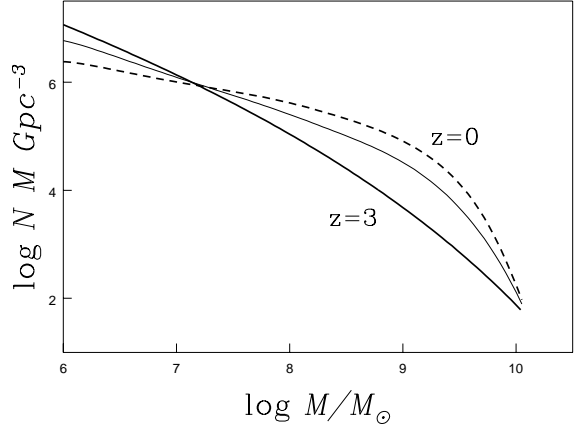


Fig. 2.— The evolution of the mass distribution $N(M, z)$ of the relic BHs. Down to $z \approx 3$ this is given by $N_{PS}(M_h, z)$ with the transformation (4B), see the thick solid line. Afterwards, the distribution is driven by host interactions as described by eq. (9). The dashed line corresponds to unconstrained accretion of 1/3 of the gas sent toward the nucleus; the thin solid line corresponds to accretion constrained by the QS feedback following eq. (11). The related local mass density amounts to $\rho_{BH} \approx 4$ and $2 \cdot 10^{14} M_\odot \text{ Gpc}^{-3}$, respectively. Cosmology/cosmogony as in fig. 1.

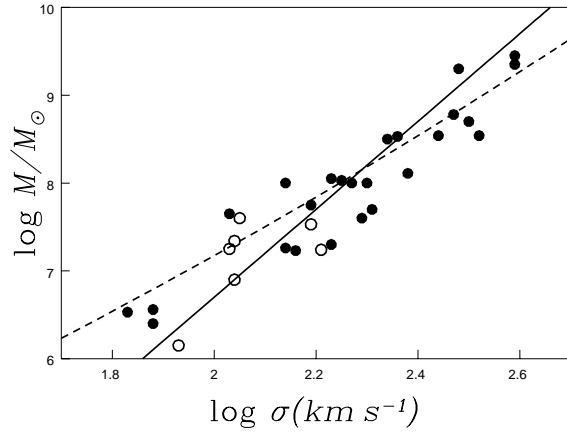


Fig. 3.— The models for the local $M - \sigma$ relation discussed in §5. Dashed: unconstrained accretion at levels $f = 5\%$, the average value given by the distribution $P(f)$. Solid: feedback constrained accretion with $\epsilon = 5 \cdot 10^{-3}$. Data from Gebhardt et al. 2000, and Ferrarese & Merritt 2000, with the open symbols marking results from reverberation mapping. The model normalizations at high M follow the fits by the same authors.